

NOTATION

$k(t)$, heat flux relaxation function; $\alpha(t)$, internal energy relaxation function; T , rod temperature; θ , ambient temperature; t , time; x , coordinate along the rod; $\sigma_{xx}(x, t)$, stress; $u(x, t)$, displacement; $\varepsilon(x, t)$, deformation; $c_0 = (E/\rho)^{1/2}$, speed of sound in the rod under isothermal conditions; E , elasticity modulus; ρ , density of the material; α_t , coefficient of thermal expansion; λ , thermal-conductivity coefficient; a , thermal-diffusivity coefficient; b , thermal-activity coefficient; $c_q = (a/\tau_T)^{1/2}$, velocity of heat propagation; τ_T , heat flux relaxation time; $\eta(t)$, unique Heaviside function.

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HYDRODYNAMICS AND HEAT AND MASS TRANSFER IN A SOLIDIFYING MELT

P. F. Zavgorodnii, F. V. Nedopekin,
and I. L. Povkh

UDC 669.18.065:532.5:536.25

The effect of natural thermal convection in a solidifying melt on the distribution of an admixture in the liquid and solid phases as a function of the Lewis and Schmidt numbers is studied numerically.

It is known [1-3] that the hydrodynamics of the melt has an important influence on the processes of heat and mass transfer in a crystallizing ingot. Therefore, in an analysis of the conditions of formation of chemical nonuniformity of ingots it is important to know the laws of the development of the convective currents and the distribution of the velocity fields in the volume of unsolidified metal.

There are several natural causes producing the mixing of the liquid metal during its crystallization:

- 1) natural thermal convection of the melt owing to its temperature nonuniformity;
- 2) shrinkage of the metal during solidification caused by the difference in densities of the liquid and solid phases;
- 3) concentration convection due to the nonuniformity of the concentration of the admixture in the melt;
- 4) the formation of nuclei of the solid phase at the crystallization front and their descent along the crystallization boundary.

One of the main causes of mixing of the melt in the process of its crystallization is natural thermal convection [1], the investigation of which is a complex and important problem.

The problem of the distribution of an admixture in a solidifying melt is solved for the most part in a one-dimensional formulation and without allowance for thermal convection, which is evidently connected with the absence of analytical methods for solving nonsteady nonlinear differential equations of transfer of an admixture in regions with moving boundaries under conditions of convective motion of the solidifying melt. In this connection the influence of natural thermal convection on the distribution of an admixture in a solidifying melt has been studied insufficiently and requires additional research.

A rectangular region, semiinfinite along the horizontal coordinate normal to the plane of the cross section, is analyzed for a study of thermal convection and its influence on the processes of transfer of an admixture in a solidifying melt. The region is filled with a melt (of low-carbon steel) with an initial temperature

Donets State University. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 33, No. 5, pp. 922-930, November, 1977. Original article submitted November 11, 1976.

higher than the crystallization temperature and with a small initial content (0.1-0.2% by weight) of a dissolved admixture (carbon).

At a time different from zero the temperature of the boundaries of the region is abruptly reduced to the crystallization temperature of the melt, as a result of which the solid phase forms at the periphery of the region near its cool walls, while in the liquid phase convective motion of the melt arises under the action of the temperature gradient. Considering the small initial content of the admixture in the melt, we assume that the solidification boundary is flat and that the density of the melt varies insignificantly as a function of the concentration of the admixture, i.e., concentration convection is ignored.

The law of advance of the solidification boundary is postulated in the form of a quadratic law, which follows from the solution of the Stefan problem with the condition of constancy of the temperature at the phase interface [4]

$$\varepsilon_i = l_i - \alpha_i \sqrt{Fo} \quad \text{and} \quad R_i = \alpha_i \sqrt{Fo} \quad (i = 1, 2),$$

where ε_i is the width of the liquid zone; R_i is the width of the solid crust; α_1 and α_2 are the coefficients of solidification.

Having defined the characteristic velocity and the characteristic pressure difference by the expressions

$$U_0 = \frac{D}{X_0}; \quad P_0 = \rho_0 U_0^2 = \rho_0 \frac{D^2}{X_0^2},$$

we write the system of Navier-Stokes equations, taken in the Boussinesq approximation, of heat transfer, mass transfer, and continuity in dimensionless form as follows:

$$\frac{\partial \bar{V}}{\partial Fo} + (\bar{V} \nabla) \bar{V} = -\nabla \pi + \varepsilon_2 \text{Sm}^2 \text{Gr} \Theta + \text{Sm} \Delta \bar{V}; \quad (1)$$

$$\frac{\partial \Theta}{\partial Fo} + (\bar{V} \nabla) \Theta = \frac{1}{\text{Lu}} \Delta \Theta; \quad (2)$$

$$\frac{\partial S}{\partial Fo} + (\bar{V} \nabla) S = \Delta S; \quad (3)$$

$$\nabla \bar{V} = 0. \quad (4)$$

The system of equations (1)-(4) is supplemented by the following boundary conditions: initial conditions

$$\bar{V}|_{Fo=0} = 0, \quad \Theta|_{Fo=0} = 1, \quad S|_{Fo=0} = 1; \quad (5)$$

boundary conditions for the velocity components

$$V_1|_{\eta_1=R_1} = V_1|_{\eta_1=\varepsilon_1} = V_1|_{\eta_2=R_2} = V_1|_{\eta_2=\varepsilon_2} = 0, \quad (6)$$

$$V_2|_{\eta_1=R_1} = V_2|_{\eta_1=\varepsilon_1} = V_2|_{\eta_2=R_2} = V_2|_{\eta_2=\varepsilon_2} = 0. \quad (7)$$

Since the melt contains a small amount of the admixture, the temperature at the solidification boundary is roughly assumed to be constant and equal to the crystallization temperature of the melt

$$\Theta|_{\eta_1=R_1} = \Theta|_{\eta_1=\varepsilon_1} = \Theta|_{\eta_2=R_2} = \Theta|_{\eta_2=\varepsilon_2} = 0. \quad (8)$$

The boundary conditions for the concentration of the admixture are written down on the basis of the equations of balance of the admixture at each boundary of the region, neglecting the diffusion of the admixture in the solid phase in comparison with that in the liquid phase

$$-\frac{\partial S}{\partial \eta_1} \Big|_{\eta_1=R_1} = R_1' (1 - k_0) S \Big|_{\eta_1=R_1}; \quad (9)$$

$$-\frac{\partial S}{\partial \eta_1} \Big|_{\eta_1=\varepsilon_1} = \varepsilon_1' (1 - k_0) S \Big|_{\eta_1=\varepsilon_1}; \quad (10)$$

$$-\frac{\partial S}{\partial \eta_2} \Big|_{\eta_2=R_2} = R_2' (1 - k_0) S \Big|_{\eta_2=R_2}; \quad (11)$$

$$-\frac{\partial S}{\partial \eta_2} \Big|_{\eta_2=\varepsilon_2} = \varepsilon_2' (1 - k_0) S \Big|_{\eta_2=\varepsilon_2}, \quad (12)$$

where $\varepsilon_i' = d\varepsilon_i/dFo$; $R_i' = dR_i/dFo$ are the rates of advance of the phase interface boundaries ($i=1, 2$). The width of the region is chosen as the characteristic size.

The problem is formulated using the Fourier diffusion time $Fo = Dt/X_0^2$, which leads to the appearance in the initial equations of the Lewis (Le) and Schmidt (Sm) numbers, whose effect on the velocity, temperature, and concentration fields in crystallizing melts is still inadequately studied.

For further transformations we introduce the curl of the velocity φ , the stream function ψ , and the new variables ζ_1 and ζ_2 which accomplish the transition from a rectangular region with moving boundaries to the region of a unit square [5]

$$V_1 = \frac{\partial \psi}{\partial \eta_2}; \quad V_2 = -\frac{\partial \psi}{\partial \eta_1}; \quad (13)$$

$$\bar{\varphi} = \text{rot } \bar{V}; \quad (14)$$

$$\zeta_1 = \frac{\eta_1 - R_1}{\varepsilon_1 - R_1}; \quad \zeta_2 = \frac{\eta_2 - R_2}{\varepsilon_2 - R_2}. \quad (15)$$

An implicit finite-difference system of variable directions (a longitudinal-transverse system [6]) is used for the numerical realization of the system of equations (1)-(4) with the boundary conditions (5)-(12) on a computer. Uniform coordinate and time (ω_h and Fo_h) grids are introduced in accordance with this. Taking the numbers of the partitions of the region along the coordinates ζ_1 and ζ_2 as equal ($J=M$), we have

$$\omega_h = \left\{ \zeta_1 = ih, \zeta_2 = mh; h = \frac{1}{J} = \frac{1}{M} > 0; \right. \\ \left. i = 0, 1, 2, \dots, J; m = 0, 1, 2, \dots, M \right\}; \quad (16)$$

$$Fo_n = \left\{ Fo = \sum_{j=0}^n \tau_j; \tau_j = A \frac{h^2}{4}; 0 < A < 1 \right\}. \quad (17)$$

With allowance for Eqs. (16) and (17) the region of the continuous argument is replaced by a region of a discrete argument, the differential operators are replaced by difference operators, and the difference analog of the boundary conditions is formulated.

Equations (1)-(4), with allowance for Eqs. (13)-(15) and using the method of fractional steps and their separation with respect to the coordinates [7], take the form

$$\frac{\bar{\varphi}_{i,m} - \varphi_{i,m}}{0.5\tau} = \frac{1}{\varepsilon_1 - R_1} \left[\frac{1}{\varepsilon_2 - R_2} \left(\frac{\partial \psi}{\partial \zeta_2} \right)_{i,m} - \zeta_1 (\varepsilon'_1 - R'_1) - R'_1 \right] \times \\ \times \left(\frac{\partial \bar{\varphi}}{\partial \zeta_1} \right)_{i,m} = \frac{Sm}{(\varepsilon_1 - R_1)^2} \left(\frac{\partial^2 \bar{\varphi}}{\partial \zeta_1^2} \right)_{i,m} - \frac{Sm^2 Gr}{\varepsilon_1 - R_1} \left(\frac{\partial \Theta}{\partial \zeta_1} \right)_{i,m}; \quad (18)$$

$$\frac{\hat{\varphi}_{i,m} - \bar{\varphi}_{i,m}}{0.5\tau} - \frac{1}{\varepsilon_2 - R_2} \left[\frac{1}{\varepsilon_1 - R_1} \left(\frac{\partial \psi}{\partial \zeta_1} \right)_{i,m} + \right. \\ \left. + \zeta_2 (\varepsilon'_2 - R'_2) + R'_2 \right] \left(\frac{\partial \hat{\varphi}}{\partial \zeta_2} \right)_{i,m} = \frac{Sm}{(\varepsilon_2 - R_2)^2} \left(\frac{\partial^2 \hat{\varphi}}{\partial \zeta_2^2} \right)_{i,m}; \quad (19)$$

$$\frac{\bar{\Theta}_{i,m} - \Theta_{i,m}}{0.5\tau} + \frac{1}{\varepsilon_1 - R_1} \left[\frac{1}{\varepsilon_2 - R_2} \left(\frac{\partial \psi}{\partial \zeta_2} \right)_{i,m} - \zeta_1 (\varepsilon'_1 - R'_1) - R'_1 \right] \times \\ \times \left(\frac{\partial \bar{\Theta}}{\partial \zeta_1} \right)_{i,m} = \frac{1}{Lu (\varepsilon_1 - R_1)^2} \left(\frac{\partial^2 \bar{\Theta}}{\partial \zeta_1^2} \right)_{i,m}; \quad (20)$$

$$\frac{\hat{\Theta}_{i,m} - \bar{\Theta}_{i,m}}{0.5\tau} - \frac{1}{\varepsilon_2 - R_2} \left[\frac{1}{\varepsilon_1 - R_1} \left(\frac{\partial \psi}{\partial \zeta_1} \right)_{i,m} - \zeta_2 (\varepsilon'_2 - R'_2) + R'_2 \right] \times \\ \times \left(\frac{\partial \hat{\Theta}}{\partial \zeta_2} \right)_{i,m} = \frac{1}{Lu (\varepsilon_1 - R_1)^2} \left(\frac{\partial^2 \hat{\Theta}}{\partial \zeta_2^2} \right)_{i,m}; \quad (21)$$

$$\frac{\bar{S}_{i,m} - S_{i,m}}{0.5\tau} + \frac{1}{\varepsilon_1 - R_1} \left[\frac{1}{\varepsilon_2 - R_2} \left(\frac{\partial \psi}{\partial \zeta_2} \right)_{i,m} - \zeta_1 (\varepsilon'_1 - R'_1) - R'_1 \right] \times \\ \times \left(\frac{\partial \bar{S}}{\partial \zeta_1} \right)_{i,m} = \frac{1}{(\varepsilon_1 - R_1)^2} \left(\frac{\partial^2 \bar{S}}{\partial \zeta_1^2} \right)_{i,m}; \quad (22)$$

$$\frac{\hat{S}_{i,m} - \bar{S}_{i,m}}{0.5\tau} - \frac{1}{\varepsilon_2 - R_2} \left[\frac{1}{\varepsilon_1 - R_1} \left(\frac{\partial \hat{\psi}}{\partial \xi_1} \right)_{i,m} + \xi_2 (\varepsilon_2' - R_2') + R_2' \right] \times \\ \times \left(\frac{\partial \hat{S}}{\partial \xi_2} \right)_{i,m} = \frac{1}{(\varepsilon_2 - R_2)^2} \left(\frac{\partial^2 \hat{S}}{\partial \xi_2^2} \right)_{i,m}; \quad (23)$$

where $\varphi = \varphi^n$; $\Theta = \Theta^n$; $S = S^n$; $\bar{\varphi} = \varphi^{n+\frac{1}{2}}$; $\bar{\Theta} = \Theta^{n+\frac{1}{2}}$; $\bar{S} = S^{n+\frac{1}{2}}$; $\hat{\varphi} = \varphi^{n+1}$; $\hat{\Theta} = \Theta^{n+1}$; $\hat{S} = S^{n+1}$.

The difference approximation of the boundary conditions is made, with the boundary conditions for the curl of the velocity and the concentration being obtained through the expansion of these functions in a Taylor series in the neighborhoods of the respective boundaries, using the initial boundary conditions (6)-(7) and (9)-(12), as well as Eqs. (3)-(4).

Initial conditions:

$$\Psi_{i,m}^0 = 0; \quad \Phi_{i,m}^0 = 0; \quad \Theta_{i,m}^0 = 1; \quad S_{i,m}^0 = 1. \quad (24)$$

Boundary conditions for the stream function:

$$\Psi_{0,m} = \Psi_{1,m} = \Psi_{i,0} = \Psi_{i,1} = 0. \quad (25)$$

Boundary conditions for the curl of the velocity:

$$\Phi_{i,0} = -\frac{2}{h^2} \Psi_{i,1}; \quad (26)$$

$$\Phi_{i,M} = \frac{2}{h^2} \Psi_{i,M-1}; \quad (27)$$

$$\Phi_{0,M} = -\frac{2}{h^2} \Psi_{1,m}; \quad (28)$$

$$\Phi_{J,m} = -\frac{2}{h^2} \Psi_{J-1,m}. \quad (29)$$

Boundary conditions for the concentration:

$$\bar{S}_{0,m} = \frac{\bar{S}_{1,m} + \frac{h^2}{\tau} S_{0,m}}{1 - hR_1'K + \frac{h^2}{2} \left(\frac{2}{\tau} + R_1'^2K \right)}; \quad (30)$$

$$\bar{S}_{J,m} = \frac{\bar{S}_{J-1,m} + \frac{h^2}{\tau} S_{J,m}}{1 + h\varepsilon_1'K + \frac{h^2}{2} \left(\frac{2}{\tau} + \varepsilon_2'^2K \right)}; \quad (31)$$

$$\hat{S}_{i,0} = \frac{\hat{S}_{i,1} + \frac{h^2}{\tau} \bar{S}_{i,0}}{1 - hR_2'K + \frac{h^2}{2} \left(\frac{2}{\tau} + R_2'^2K \right)}; \quad (32)$$

$$\hat{S}_{i,M} = \frac{\hat{S}_{i,M-1} + \frac{h^2}{\tau} \bar{S}_{i,M}}{1 + h\varepsilon_2'K + \frac{h^2}{2} \left(\frac{2}{\tau} + \varepsilon_2'K \right)}; \quad (33)$$

where $K = 1 - k_0$.

As the method of constructing the difference systems, on the basis of the accuracy requirements, we use the integrointerpolation method developed by Samarskii [6], as a result of which the system of equations (18)-(23) and the boundary conditions (24)-(33) are reduced to a system of trial-run equations which is realized on a Dnepr-21 computer.

From the conditions of mathematical stability and sufficiently high computation accuracy a spatial grid 32×32 in size with $A = 0.0008$ is established as a result of a numerical experiment.

The influence of the Lewis and Schmidt numbers on the processes of heat and mass transfer in a solidifying ingot is studied for a melt of low-carbon steel with the following parameters: $Gr = 0.2 \cdot 10^7$; $k_0 = 0.5$; $\alpha_1 = \alpha_2 = 10$; $L_2 = 3$.

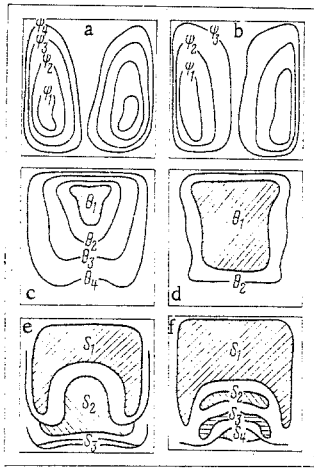


Fig. 1

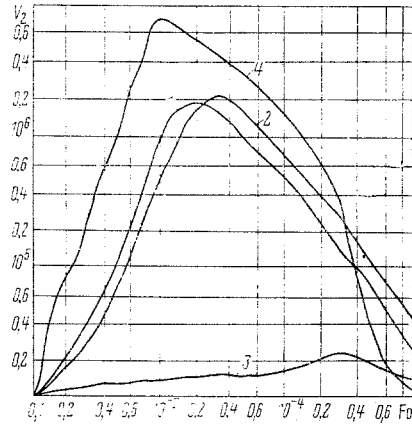


Fig. 2

Fig. 1. Influence of Lewis numbers on isolines of the stream functions (a and b), on the temperature field (c and d), and on the concentration field (e and f) at the time $Fo = 0.32 \cdot 10^{-4}$ for a Schmidt number $Sm = 90$: a) $Le = 0.24 \cdot 10^{-2}$, $\psi_1 = 0.8 \cdot 10^4$, $\psi_2 = 0.5 \cdot 10^4$, $\psi_3 = 0.3 \cdot 10^4$, $\psi_4 = 0.1 \cdot 10^4$; b) $Le = 0.012$, $\psi_1 = 0.42 \cdot 10^4$, $\psi_2 = 0.26 \cdot 10^4$, $\psi_3 = 0.7 \cdot 10^3$; c) $Le = 0.24 \cdot 10^{-2}$, $\theta_1 = 0.99$, $\theta_2 = 0.97$, $\theta_3 = 0.84$, $\theta_4 = 0.5$; d) $Le = 0.012$, $\theta_1 = 0.99$, $\theta_2 = 0.94$; e) $Le = 0.24 \cdot 10^{-2}$, $S_1 = 1$, $S_2 = 1.06$, $S_3 = 1.09$; f) $Le = 0.012$, $S_1 = 1$, $S_2 = 1.02$, $S_3 = 1.1$, $S_4 = 1.15$.

Fig. 2. Variation in the time of the maximum value of the velocity component V_2 for the descending current as a function of the Lewis and Schmidt numbers: 1) $Le = 0.24 \cdot 10^{-2}$; $Sm = 90$; 2) $Le = 0.012$; $Sm = 90$; 3) $Sm = 9$; $Le = 0.24 \cdot 10^{-2}$; 4) $Sm = 900$; $Le = 0.24 \cdot 10^{-2}$.

The authors discussed the influence of the Grashof numbers on these processes earlier [5].

An analysis of Fig. 1 shows that with an increase in the Lewis number one observes a slowing tendency in the development of the velocity field (Fig. 1a and b) and accordingly in that of the thermal (Fig. 1c and d) and concentration (Fig. 1e and f) fields. In the process an increase occurs in the time segment of the acceleration to the maximum value (Fig. 2), which remains approximately the same for all Lewis numbers in the interval studied ($Le = 0.0024 - 0.012$).

This property of the development of thermal convection with variation in the Lewis number is explained by the fact that melts with a low thermal conductivity correspond to large Lewis numbers, as a rule, so that the time of establishment of the temperature gradient, which determines the maximum velocity, increases.

From an analysis of the concentrations of the admixture at the phase transition boundary (see Table 1) it follows that an increase in the Lewis number has an effect on the distribution of the admixture similar to the effect of the Grashof number [5], i.e., with an increase in the Lewis number one observes a decrease in the concentration of the admixture at the phase interface up to a time corresponding to $\sim 50\%$ solidification of the liquid melt. The explanation for this fact is that large Lewis numbers correspond to large velocities of the convection currents (Fig. 2, curves 1 and 2), not counting the initial segment, and this is the main reason for the strong withdrawal of the admixture from the phase interface into the interior of the melt.

The concentration S_1 of the admixture in the solid phase is determined from the following equilibrium equation which is satisfied at the boundary of the phase transition

$$S_{1|_b} = k_0 S|_b \quad (34)$$

A study of the distribution of the impurity in the solid phase (Fig. 3a) indicates that with an increase in Le the peripheral sections of the crystallized melt are depleted of the admixture and the axial zone of the ingot is enriched with the admixture. This is explained by the described character of the behavior of the admixture at the boundary of the phase transition as a function of Le . Thus, an increase in the Lewis number leads to strengthening of the nonuniformity of the distribution of the admixture over the height of the ingot and to its more uniform distribution over horizontal cross sections of the solidifying melt.

TABLE 1. Concentrations of Admixture at Phase Interface for Different Lewis and Schmidt Numbers with $\eta_2 = 1.5$

R_1	S_b			
	$S_m = 90$		$Le = 0.24 \cdot 10^{-2}$	
	$Le = 0.24 \cdot 10^{-2}$	$Le = 0.012$	$S_m = 9$	$S_m = 900$
0,1	1,44	1,3	1,8	1,28
0,2	2	1,84	2,2	2
0,3	2,56	2,47	2,34	2,54
0,4	2,56	2,65	2,36	2,56
0,5	2,9	3	2,68	3,06

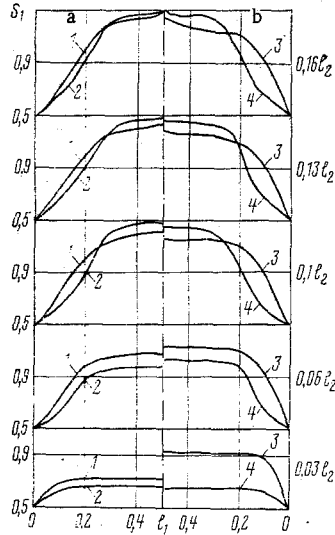


Fig. 3

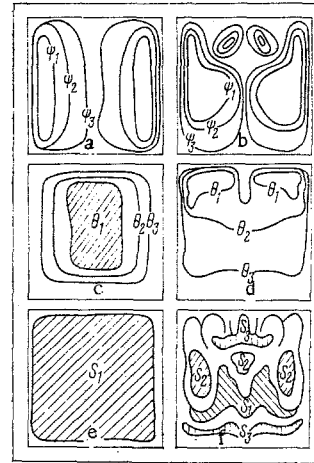


Fig. 4

Fig. 3. Distribution of the admixture in the solid phase as a function of the Lewis (a) and Schmidt (b) numbers: 1) $Le = 0.24 \cdot 10^{-2}$; 2) $Le = 0.012$; 3) $S_m = 9$; 4) $S_m = 900$.

Fig. 4. Influence of Schmidt numbers on isolines of the stream function (a and b), on the temperature field (c and d), and on the concentration field (e and f) at the time $Fo = 0.32 \cdot 10^{-4}$ for the Lewis number $Le = 0.24 \cdot 10^{-2}$: a) $S_m = 9$, $\psi_1 = 0.29 \cdot 10^2$, $\psi_2 = 0.19 \cdot 10^2$, $\psi_3 = 0.14 \cdot 10^1$; b) $S_m = 900$, $\psi_1 = 0.14 \cdot 10^5$, $\psi_2 = 0.1 \cdot 10^5$, $\psi_3 = 0.9 \cdot 10^3$; c) $S_m = 9$, $\theta_1 = 0.99$, $\theta_2 = 0.98$, $\theta_3 = 0.79$; d) $S_m = 900$, $\theta_1 = 0.55$, $\theta_2 = 0.49$, $\theta_3 = 0.2$; e) $S_m = 9$, $S_1 = 1$; f) $S_m = 900$, $S_1 = 1$, $S_2 = 1.04$, $S_3 = 1.09$.

The second part of the investigations is devoted to a study of the processes of heat and mass transfer in a solidifying melt under the conditions of natural thermal convection as a function of the Schmidt number for a Lewis number $Le = 0.0024$.

An increase in the Schmidt number promotes the acceleration of the development of thermal convection (Fig. 4a, b), with the time segment of the acceleration of the velocity to the maximum value decreasing and the value of the velocity of convective motion of the melt increasing (Fig. 2, curves 3 and 4). The strengthening of thermal convection in the solidifying melt with an increase in S_m leads to intensification of the processes of heat transfer (Fig. 4c, d) and mass transfer (Fig. 4e, f) in the liquid core of the region under study.

The behavior of the concentration of the admixture at the boundary of the phase transition (see Table 1) with an increase in the Schmidt number is similar to the behavior of the analogous concentration with an increase in the Grashof [5] and Lewis numbers, i.e., one observes a decrease in the concentration of the admixture in the first half of the solidification process and its increase at later times.

The described character of the distribution of the admixture at the boundary of the phase transition can be explained by the fact that up to the time corresponding to the solidification of $\sim 50\%$ of the melt, the thermal

convection strengthens the transfer of the admixture from the crystallization front into the interior of the melt. And the intensification of the process of withdrawal of the admixture from the solidification boundary is the main reason for its decrease at the latter.

The increase in the concentration of the admixture following its decrease in comparison with the case when thermal convection is less intense ($Sm=9$) is obviously explained by the general saturation of the liquid core by the admixture, which takes place much more intensively for a melt with $Sm=900$ than for a melt with $Sm=9$.

An analysis of the graphs of the distribution of the admixture in the solid phase (Fig. 3b) for the case of an equilibrium coefficient (34) shows that with an increase in the Schmidt number the central part of the crystallized melt is enriched with the admixture owing to the depletion of the peripheral sections.

Thus, the patterns of behavior of the admixture in a solidifying melt with variation in the Lewis, Schmidt, and Grashof numbers are similar in a qualitative respect, and therefore the conclusion drawn in [5] that the required degree of purification of an industrial ingot from an admixture can be achieved by controlling the intensity of thermal convection in the liquid core of the crystallizing melt is confirmed by the results of the studies presented in the present report.

NOTATION

X_0 , U_0 , P_0 , characteristic size, velocity, and pressure; D , coefficient of diffusion of admixture in the liquid phase; ρ_0 , density of melt at the crystallization temperature; $\vec{V} = \vec{U}/U_0$, dimensionless velocity vector; $\pi = P/P_0$, dimensionless pressure; \vec{e}_2 , unit vector coinciding with the direction of free-fall acceleration; $Sm = \nu/D$, Schmidt number; ν , kinematic viscosity of the melt; $Gr = q(T_0 - T_C)\beta X_0^3/\nu^2$, Grashof number; q , absolute value of free-fall acceleration vector; T_0 , T_C , initial temperature of melt and its crystallization temperature; β , coefficient of volumetric expansion of liquid melt; $\Theta = (T - T_C)/(T_0 - T_C)$, dimensionless temperature of the melt; T , dimensional temperature of the melt; $Fo = tD/X_0^2$, Fourier number or dimensionless time; $Le = D/a$, Lewis number; a , coefficient of thermal diffusivity of the melt; $S = C/C_0$, relative concentration of admixture in the melt; C , C_0 , absolute and initial concentration of admixture in the melt; V_1 , V_2 , horizontal and vertical velocity components; $\eta_1 = X_1/X_0$, $\eta_2 = X_2/X_0$, dimensionless horizontal and vertical coordinates; X_1 , X_2 , dimensional coordinates; k_0 , equilibrium coefficient of distribution of the admixture.

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